Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet for 2021-11-17

Computations

Problem 1. Let E be the solid cylinder $x^2 + y^2 \leq 4, 1 \leq z \leq 3$. Let $\mathbf{F} = \langle 0, 0, z \rangle$. Compute $\iint_{\partial E} \mathbf{F} \cdot \mathrm{d}\mathbf{S}$, where ∂E denotes the boundary surface of E oriented outwards (note that ∂E consists of three faces).

Once you learn the Divergence Theorem, you will have another way of doing this problem.

Problem 2. Use Stokes' Theorem to compute

$$\int_C \langle x^2 y, \frac{1}{3}x^3, xz \rangle \cdot \mathrm{d}\mathbf{r}$$

where C is the curve

$$x = \cos t,\ y = \sin t,\ z = \cos 2t,\ 0 \leq t \leq 2\pi.$$

(First find an oriented surface S such that $\partial S = C$.)

Problem 3. The gravitational vector field (generated by a point mass at the origin) is proportional to

$$\mathbf{F} = -r^{-3}\mathbf{r} = \frac{-\langle x, y, z \rangle}{(x^2 + y^2 + z^2)^{3/2}}.$$

(a) Let S denote the sphere $x^2 + y^2 + z^2 = 1$, oriented outwards. Compute $\iint_S \mathbf{F} \cdot \mathrm{d}\mathbf{S}$.

(b) Explain why there does not exist a vector field $\mathbf{G}$ with the same domain as $\mathbf{F}$ such that $\nabla \times \mathbf{G} = \mathbf{F}$.
Hint: Suppose for the sake of contradiction that such a $\mathbf{G}$ existed. What happens if you apply Stokes' Theorem to the integral from (a)?

Problem 4. If C is allowed to be any simple closed curve on the surface $z = xy$ that is oriented **clockwise** when viewed from above, what choice of C maximizes the integral

$$\int_C \langle yz, xz + yz - 2z, xy + x^2/2 \rangle \cdot \mathrm{d}\mathbf{r}$$

and what is that maximum value?